Non-renewable resources: Are we back to zero growth?

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Economic growth: Theory and Empirical Methods

Introduction

- Constant growth in output per worker in the Solow model depends on the assumption that non-labor factors of production can be increased indefinitely.
- You have already discussed the issue of finite natural resources:
  - We will see that constant growth is still a likely outcome.
  - Moreover, price data suggests that seemingly necessary and finite resources are either not necessary or not finite.
- You have also discussed the issue of pollution and the environmental Kuznets curve:
  - We will see that technological progress again gives hope for long-run economic growth.
  - We will explain the environmental Kuznets curve by transition dynamics.

Non-renewable resources

History

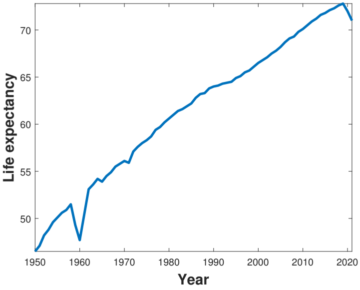
Meadows et al. (1972) in their contribution for the *Club of Rome*, conducted computer simulations for world output and population. They emphasized particularly the finite amount of some key resources which would make their use less and less feasible:

"Given present resources consumption rates and the projected increase in the rates, the great majority of the currently important nonrenewable resources will be extremely costly 100 years from now. [...] The prices of those resources with the shortest static reserve indices have already begun to increase. The price of mercury, for example, has gone up 500 percent in the last 20 years; the price of lead has increased 300 percent in the last 30 years."

History II

Ehrlich (1968) revived the Malthusian logic of a population growing faster than food supply writing:

"The battle to feed all of humanity is over. In the 1970s and 1980s hundreds of millions of people will starve to death [...]. At this late date nothing can prevent a substantial increase in the world death rate."



Source: United Nations

Overview

- We are going to introduce a non-renewable resource into the Solow model.
- You can think of oil, gas, minerals, and other things that are in finite supply but important in production.
- The key difference to capital is that these resources will be used-up over time.
- Note, this is also different from land in the Malthus model which was finite but fixed.

Production

Assume production is given by

$$Y(t) = A(t)^{1-\alpha}K(t)^{\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma}, \quad (1)$$

where $E(t)$ is the amount of the non-renewable resource used in production. Note, the function has constant returns to scale in $K(t), E(t), L(t)$. As in the Solow model, there are different (but economically equivalent) ways to have $A(t)$ in the production function. Here, it enters with the same exponent as in the basic Solow model which will make the comparison simpler.

Factor dynamics

As before, we have

$$\frac{\dot{L}(t)}{L(t)} = n, \tag{2}$$

$$\frac{\dot{A}(t)}{A(t)} = g, \tag{3}$$

$$\dot{K}(t) = sY(t) - \delta K(t). \tag{4}$$

Dynamics of the non-renewable resource

Assume we start in period 0 with a stock of the non-renewable resource $R(0)$. We have that our use of the resource depletes its stock:

$$\dot{R}(t) = -E(t). \tag{5}$$

One can show that when competitive firms own the resource, optimal behavior implies that each period a constant fraction of the remaining stock is used:

$$s_E = \frac{E(t)}{R(t)}. \tag{6}$$

Hence, the stock must decline over time at rate s_E:

$$\frac{\dot{R}(t)}{R(t)} = -s_E = \frac{\dot{E}(t)}{E(t)}. \tag{7}$$

Dynamics of the non-renewable resource II

$$\frac{\dot{R}(t)}{R(t)} = -s_E. \tag{8}$$

We know the solution to this differential equation:

$$R(t) = R(0)\exp(-s_E t). \tag{9}$$

That is, the stock is declining exponentially over time. Finally, as $E(t) = s_E R(t)$, we know that consumption of the resource is declining exponentially over time:

$$E(t) = s_E R(0)\exp(-s_E t). \tag{10}$$

The steady state

We begin again with analyzing behavior in steady state. As before, we first find the capital-to-output ratio in steady state:

$$z(t) = \frac{K(t)}{Y(t)} = \frac{K(t)^{1-\alpha}}{A(t)^{1-\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma}} \tag{11}$$

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)g + \gamma s_E - (1-\alpha-\gamma)n. \tag{12}$$

Hence, in steady state,

$$0 = \left(\frac{\dot{K}(t)}{K(t)}\right)^{*} - g + \frac{\gamma}{(1-\alpha)}s_E - \frac{(1-\alpha-\gamma)}{(1-\alpha)}n. \tag{13}$$

The steady state capital-to-output ratio

From the capital-accumulation equation, we have

$$\dot{K}(t) = sA(t)^{1-\alpha}K(t)^{\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma} - \delta K(t) \tag{14}$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{s}{z(t)} - \delta \tag{15}$$

Putting things together,

$$n + g - \frac{\gamma}{(1-\alpha)}(n + s_E) = \frac{s}{z^*} - \delta \tag{16}$$

$$z^* = \frac{s}{n + g + \delta - \frac{\gamma}{(1-\alpha)}(n + s_E)}, \tag{17}$$

which is indeed constant.

The steady state capital-to-output ratio II

$$z^* = \left(\frac{K(t)}{Y(t)}\right)^* = \frac{s}{n + g + \delta - \frac{\gamma}{(1-\alpha)}(n + s_E)} \tag{18}$$

The result is the basic Solow model with $\gamma = 0$. With $\gamma > 0$ (and $n + s_E > 0$), the ratio is higher than in the standard Solow model. The reason is that capital growth is lower:

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^* = n + g - \frac{\gamma}{(1-\alpha)}(s_E + n). \tag{19}$$

Output in steady state

We need to rewrite the production function in terms of the capital output ratio:

$$Y(t) = A(t)^{1-\alpha} K(t)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma} \tag{20}$$

$$Y(t)^{1-\alpha} = A(t)^{1-\alpha} \left(\frac{K(t)}{Y(t)}\right)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma} \tag{21}$$

$$Y(t) = A(t) \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} E(t)^{\frac{\gamma}{1-\alpha}} L(t)^{1-\frac{\gamma}{1-\alpha}} \tag{22}$$

$$Y(t) = A(t) \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \left(s_E R(0) \exp(-s_E t)\right)^{\frac{\gamma}{1-\alpha}} L(t)^{1-\frac{\gamma}{1-\alpha}} \tag{23}$$

Output per worker in steady state

$$y(t)^* = \left(\frac{s}{n + g + \delta - \frac{\gamma}{(1-\alpha)}(n + s_E)} \right)^{\frac{\alpha}{1-\alpha}} \\ (s_E R(0) \exp(-s_E t))^{\frac{\gamma}{1-\alpha}} L(t)^{-\frac{\gamma}{1-\alpha}} A(t) \quad (24)$$

Note, the depletion rate s_E enters three into the expression. A higher depletion rate (i) increases the capital-to-output ratio, (ii) raises the resource use and, thereby, production, and (iii) reduces the stock of resources over time and, thereby the resource use.

Output growth in steady state

Taking logs and the derivative with respect to time yields output growth:

$$Y(t) = A(t)\left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \left(s_E R(0)\exp(-s_E t)\right)^{\frac{\gamma}{1-\alpha}} L(t)^{1-\frac{\gamma}{1-\alpha}}$$

$$\ln Y(t)^* = \ln A(t) + \frac{\alpha}{1-\alpha}\ln\left(\frac{K(t)}{Y(t)}\right)^* + \frac{\gamma}{1-\alpha}\left(\ln(s_E R(0)) - s_E t\right) + \left(1 - \frac{\gamma}{1-\alpha}\right)\ln L(t)$$

$$\left(\frac{\dot{Y}(t)}{Y(t)}\right)^* = g + n - \frac{\gamma}{1-\alpha}(s_E + n).$$

Output growth in steady state II

$$\left(\frac{\dot{Y}(t)}{Y(t)}\right)^* = g + n - \frac{\gamma}{1-\alpha}(s_E + n).$$

- The depletion rate acts like negative technological progress on growth as the non-renewable resource becomes more scarce over time.
- Labor contributes to growth with a rate $< n$. Due to the fixed factor, as in the Malthus model, population growth reduces worker's productivity over time.

Output per worker growth in steady state

Instead of total output, we can also look at output per capita:

$$y(t) = \frac{Y(t)}{L(t)} = A(t)\left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \left(s_E R(0)\exp(-s_E t)\right)^{\frac{\gamma}{1-\alpha}} L(t)^{-\frac{\gamma}{1-\alpha}}$$

$$\left(\frac{\dot{y}(t)}{y(t)}\right)^{*} = g - \frac{\gamma}{1-\alpha}(s_E + n).$$

The depletion rate has the same negative effect on output per capita as the population growth rate. Both reduce the efficiency of labor over time. We have positive growth in GDP per capita iff

$$g > \frac{\gamma}{1-\alpha}(s_E + n).$$

Dynamics of the capital to output ratio

We now turn to out of steady state dynamics. As in the standard Solow model, the capital to output ratio is key to study the dynamics of the model. We have already derived:

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)(g+n) + \gamma(s_E + n)$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{s}{z(t)} - \delta$$

Hence, we have

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\left[\frac{s}{z(t)} - \delta\right] - (1-\alpha)(g+n) + \gamma(s_E + n).$$

Dynamics of the capital to output ratio II

Rewriting, we obtain

$$\dot{z}(t) = (1-\alpha)s - \left[\underbrace{(1-\alpha)[\delta + g + n] - \gamma(n + s_E)}_{\phi}\right] z(t)$$

Defining $u(t) = \dot{z}(t) = (1-\alpha)s - \phi z(t)$:

$$\Rightarrow \dot{u}(t) = -\phi \dot{z}(t)$$
$$\Rightarrow \dot{u}(t) = -\phi u(t)$$
$$u(t) = u(0)\exp(-\phi t).$$

Dynamics of the capital to output ratio III

Substituting back gives

$$(1-\alpha)s - \phi z(t) = [(1-\alpha)s - \phi z(0)]\exp(-\phi t)$$
$$\frac{K(t)}{Y(t)} - \frac{s}{\kappa} = \left[\frac{K(0)}{Y(0)} - \frac{s}{\kappa}\right]\exp(-\phi t)$$
$$\phi = (1-\alpha)[\delta + g + n] - \gamma(n + s_E)$$
$$\kappa = \delta + g + n - \frac{\gamma}{1-\alpha}(n + s_E)$$

- $\frac{K(t)}{Y(t)} - \left(\frac{K(t)}{Y(t)}\right)^*$ converges to zero at rate ϕ.
- Hence, s_E not only increases the steady state capital-to-output ratio but also slows down convergence to steady state.

Output per worker growth

$$y(t) = \frac{Y(t)}{L(t)} = A(t) \left(\frac{K(t)}{Y(t)} \right)^{\frac{\alpha}{1-\alpha}} (s_E R(0) \exp(-s_E t))^{\frac{\gamma}{1-\alpha}} L(t)^{-\frac{\gamma}{1-\alpha}}$$

$$\frac{\dot{y}(t)}{y(t)} = g - \frac{\gamma}{1-\alpha}(s_E + n) + \frac{\alpha}{1-\alpha} \frac{\dot{z}(t)}{z(t)}.$$

- As in the Solow model, output per worker only grows quicker than in steady state if the capital to output ratio is growing.
- Increasing the extraction rate slows down long run growth by $\frac{\gamma}{1-\alpha}$.
- However, the growth rate is initially higher than the new long run growth rate as the capital to output ratio grows.

The steady state price of non-renewables over time

Given our Cobb-Douglas production function, the share of income going to non-renewables should be constant over time:

$$P_E(t)E(t) = \gamma Y(t)$$
$$P_E(t) = \gamma \frac{Y(t)}{E(t)}.$$

Take logs and the derivative with respect to time gives the growth rate in non-renewable prices:

$$\frac{\dot{P}_E(t)}{P_E(t)} = \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{E}(t)}{E(t)}$$
$$\frac{\dot{P}_E(t)}{P_E(t)} = g - \frac{\gamma}{1-\alpha} s_E + \left(1 - \frac{\gamma}{1-\alpha}\right) n + s_E$$
$$\frac{\dot{P}_E(t)}{P_E(t)} = g + \left(1 - \frac{\gamma}{1-\alpha}\right)(n + s_E).$$

The steady state price of non-renewables over time II

$$\frac{\dot{P}_E(t)}{P_E(t)} = g + \left(1 - \frac{\gamma}{1-\alpha}\right)(n + s_E)$$
$$\frac{\dot{P}_E(t)}{P_E(t)} = g + \frac{1-\alpha-\gamma}{1-\alpha}(n + s_E) > 0$$

The price of non-renewables rises over time for three reasons:

- Technological progress makes non-renewables more productive over time.
- Population growth makes non-renewables more productive over time.
- The falling stock of non-renewables makes them more productive over time.

The steady state price of non-renewables relative to labor

To compare the predictions of the model, it is simpler to look at relative prices. Given constant factor shares, we have:

$$\frac{P_E(t)E(t)}{w(t)L(t)} = \frac{\gamma Y(t)}{(1-\gamma-\alpha)Y(t)}$$

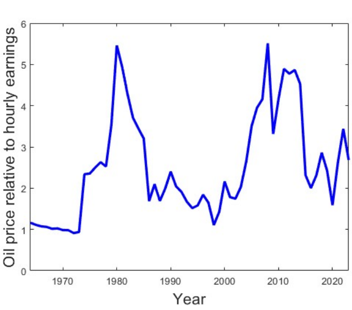
$$\frac{P_E(t)}{w(t)} = \frac{\gamma}{(1-\gamma-\alpha)}\frac{L(t)}{E(t)} = \frac{\gamma}{(1-\gamma-\alpha)}\frac{L(0)\exp(nt)}{s_E R(0)\exp(-s_E t)}$$

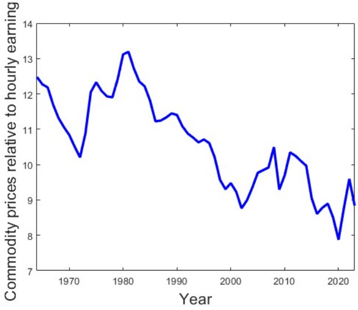
Next, take logs and the derivative with respect to time to get the growth rate in the price wage ratio, $RP(t) = \frac{P_E(t)}{w(t)}$:

$$\frac{\dot{R}P(t)}{RP(t)} = n + s_E.$$

With $n > 0$, resources become more scarce over time relative to labor implying that their relative price is growing.

Price of commodities

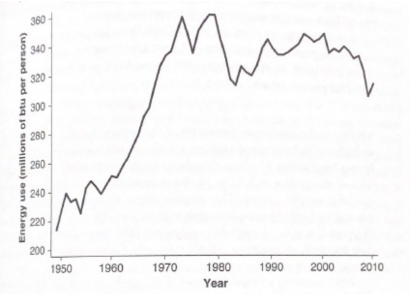




Source: St. Louis Fed

Instead of rising prices for non-renewables relative to wages, we have, if any, falling prices.

Consumption of commodities



Also, we do not observe a slow-down in the use of non-renewables.

Why the predictions fail

Simon (1980) provides a good example from history:

In the 16th century, most ships were build out of wood leading to deforestation of large parts in Europe.

⇒ The price of wood rose leading to incentives to innovate by using other materials.

⇒ Over time, ships were build out of iron and later steel. Moreover, we invented ways to recycle these resources.

Why the predictions fail II

Critiques may reply that, ultimately, the total stock of non-renewable resources is finite. However,

- practically, some resources are close to infinite, they just become more expensive to mine (at the current technology).
  - The amount of proven oil reserves doubled between 1980 and 2009.
  - So far, we mined 700 million metric tons of copper. Estimates are that 6.3 billion are still in the earth crust. Next, we may go to space.

- at a higher level of abstraction, physics tells us that we do not use-up anything, we simply transform material into other material that is of more use to us. How good we are in this depends on our technology, i.e., recipes. Accordingly, Simon (1980) identifies human ingenuity as the ultimate resource.

Mineral resources

Mineral	1950 Reserves	Production 1950–2000	2000 Reserves
Tin	6	11	10
Copper	100	339	340
Iron Ore	19,000	37,583	140,000
Lead	40	150	64
Zinc	70	266	190

Source: Blackman and Baumol (2008)

Pollution

A green Solow Model

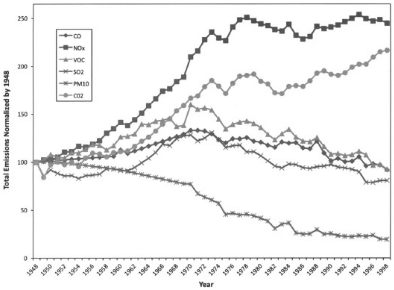
- One may think of the environment as a non-renewable resource. As pollution increases, the resource becomes depleted.
- Pollution may be best thought of something we can pay resources for to avoid it:
  - Use production technologies that create less pollution (energy) but are more expensive.
  - Create energy from green, expensive sources.
- Brock and Taylor (2010) present data on pollution and a model to understand the data.

Data facts about pollution

Brock and Taylor (2010) highlight 3 data facts about pollution:

1. Pollution increases initially with per capita income but starts falling at some point, an environmental Kuznets Curve.
2. Pollution per unit produced falls over time.
3. Abatement costs are a small, constant share of national output.

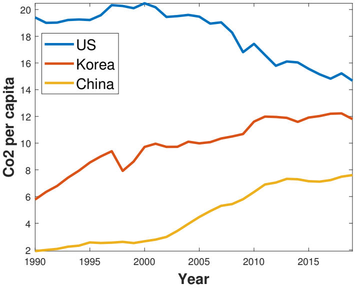
An environmental Kuznets Curve



Source: Brock and Taylor (2010)

In the US, despite income growth, most pollutant emissions are falling since 1984.

An environmental Kuznets Curve II



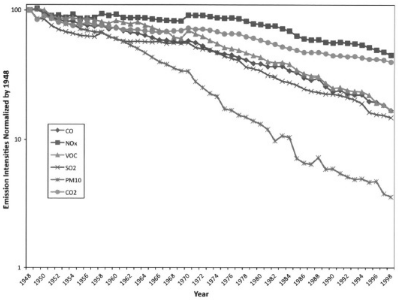
Source: World Bank

Since 2000, this is also true for CO_2 emissions. Poorer countries still increase their emission levels.

An environmental Kuznets Curve III

In 1952 and 1969, a section of the Cuhayoga river in Ohio was so covered in oil that it caught fire. The latter incident contributed to amendments to the Clean Water Act and the founding of the federal Environmental Protection Agency.

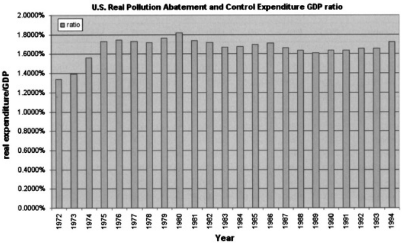
Falling emission intensity



Source: Brock and Taylor (2010)

- Emissions per units produced are falling over time for a large variety of emissions.
- The growth rate is close to constant over time.

Constant cost share of abatement



Source: Brock and Taylor (2010)

Since 1975, the abatement costs as share of GDP have been constant around 1.7%.

What are the implications from this data?

You may think that as we become richer, we can divert more resources to abatement, i.e., the environment is a luxury good. But

- this would imply that the cost share of abatement should rise as we become richer.

Instead, a constant cost share with increasing abatement suggests that we become more productive over time in abatement.

- We develop less resource-intensive production technologies.
- We switch to goods that are less resource intensive.

A model of emissions over time

- We are now ready to think about a model of emissions over time.
- The production side and capital accumulation side are as in the Solow model.
- We add to this that production creates pollution.
- We can undergo abatement to reduce the pollution but this reduces our consumption.
- Improvements in the abatement technology are the key for long-run emission dynamics.

Production and capital accumulation

We use again a Cobb-Douglas production function:

$$Y(t) = K(t)^{\alpha} \left(A(t)L(t)\right)^{1-\alpha} \tag{25}$$

with our familiar laws of motions:

$$\frac{\dot{L}(t)}{L(t)} = n \tag{26}$$

$$\frac{\dot{A}(t)}{A(t)} = g. \tag{27}$$

Pollution and abatement

Each unit of output Y creates Ω units of pollution. How much of this pollution is emitted depends on the total amount of abatement:

$$E(t) = Y(t)\Omega(t) - \Omega(t)B(t). \tag{28}$$

We assume that the abatement technology, B, is a constant returns to scale function depending on total output and the effort we put into abatement, $\theta Y(t)$:

$$B(t) = B(Y(t), \theta Y(t)). \tag{29}$$

Idea:

- The more we pollute, i.e., produce, the more we can reduce emissions.
- The amount we reduce pollutants depends on our effort.

Emissions over time

$$E(t) = Y(t)\Omega(t) - \Omega(t)B(Y(t), \theta Y(t)) \tag{30}$$

$$E(t) = Y(t)\Omega(t)\left[1 - B(1, \theta)\right] \tag{31}$$

Hence, emissions per unit produced are

$$\frac{E(t)}{Y(t)} = \Omega(t)\left[1 - B(1, \theta)\right] \tag{32}$$

We have seen that, in the data, $\frac{E(t)}{Y(t)}$ is decreasing at a constant rate, and θ is constant. Hence, to match the data, we need $\Omega(t)$ to grow at a negative rate:

$$\Omega(t) = \Omega(0)\exp(-g_B t). \tag{33}$$

The national income identity, and capital dynamics

Given that we use $\theta Y(t)$ on abatement, we have for consumption and investment:

$$I(t) + C(t) = (1-\theta)Y(t). \tag{34}$$

Hence, the law of motion for capital is

$$\dot{K}(t) = (1-\theta)sK(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} - \delta K(t) \tag{35}$$

Steady state capital-to-output ratio

As before, to analyze the steady state, we derive the capital-to-output ratio:

$$z(t) = \frac{K(t)}{Y(t)} = \frac{K(t)}{K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha}} \tag{36}$$

$$= \left(\frac{K(t)}{A(t)L(t)}\right)^{1-\alpha}. \tag{37}$$

implying that the growth rate is

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)\left(n+g\right) \tag{38}$$

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^{*} = n+g \tag{39}$$

Rewriting the capital accumulation equation

The capital accumulation equation is now slightly different from the standard Solow model

$$\dot{K}(t) = s(1-\theta)K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} - \delta K(t) \tag{40}$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{s(1-\theta)}{z(t)} - \delta \tag{41}$$

Combining the equations and imposing a steady state:

$$n + g = \frac{s(1-\theta)}{z^*} - \delta \tag{42}$$

$$z^* = \left(\frac{K(t)}{Y(t)}\right)^* = \frac{s(1-\theta)}{n+g+\delta}. \tag{43}$$

Solving for the steady state

$$z^* = \left(\frac{K(t)}{Y(t)}\right)^* = \frac{s(1-\theta)}{n+g+\delta}. \tag{44}$$

Hence, output per worker and consumption per worker in steady state are

$$y(t)^* = \left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t) \tag{45}$$

$$c(t)^* = (1-s)(1-\theta)\left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t) \tag{46}$$

Steady state consumption and abatement effort

$$y(t)^* = \left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t) \tag{47}$$

$$c(t)^* = (1-s)(1-\theta)\left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t) \tag{48}$$

A higher abatement effort reduces consumption per worker for two reasons:

- after abatement, less output is left over for consumption.
- higher abatement reduces capital investment and, hence, the capital-to-output ratio and, thereby, output per worker.

Pollution growth in steady state

We can derive the growth rate of output as in the standard Solow model

$$Y(t) = \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} A(t)L(t) \tag{49}$$

$$\Rightarrow \left(\frac{\dot{Y}(t)}{Y(t)}\right)^{*} = n + g. \tag{50}$$

From $E(t) = Y(t)\Omega(t)\left[1 - B(1,\theta)\right]$ we have in steady state

$$g_E^* = \frac{\dot{E}(t)}{E(t)} = n + g - g_B. \tag{51}$$

Whether total emissions fall in steady state depends on the race between output growth and the growth rate of abatement improvements. The U.S. data suggests that in steady state, total emissions fall, i.e., $g_B > n + g$.

Pollution growth outside the steady state

Again, we, first, need to know output growth

$$Y(t) = \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} A(t)L(t) \tag{52}$$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)} + g + n. \tag{53}$$

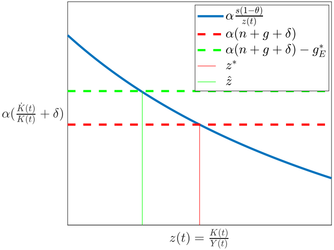
and, hence,

$$E(t) = Y(t)\Omega(t)\left[1 - B(1,\theta)\right] \tag{54}$$

$$\frac{\dot{E}(t)}{E(t)} = g_E^* + \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)}. \tag{55}$$

When the capital-to-output ratio grows, output grows and, thus, emissions grow faster than in steady state.

The environmental Kuznets curve



- At z^*, $\alpha \frac{s(1-\theta)}{z(t)} = \alpha(\delta + n + g)$, we are in steady state, and $\frac{\dot{E}(t)}{E(t)} = g_E^*$.
- At $\hat{z}$, $\alpha \frac{s(1-\theta)}{z(t)} - \alpha(\delta + n + g) = \frac{\alpha}{1-\alpha} \frac{\dot{z}(t)}{z(t)} = -g_E^*$, i.e., emission growth is zero.
- At any point to the left of $\hat{z}$, emission growth is positive, to the right, it is negative.
- As poor countries converge to steady state, their emission growth is falling.

Convergence to steady state

It is straight forward to show that

$$z(t) = \frac{s(1-\theta)}{n+g+\delta} + \left[z(0) - \frac{s(1-\theta)}{n+g+\delta}\right]\exp(-\beta t) \tag{56}$$

$$\beta = (1-\alpha)(n+g+\delta). \tag{57}$$

As in the Solow model, the growth rate of the capital-to-output ratio is fastest the further is an economy below its steady state. As

$$\frac{\dot{E}(t)}{E(t)} = g_E^* + \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)}, \tag{58}$$

emission growth will be fastest for economies well below their steady states and will slow down over time as output growth slows down.

Convergence to steady state II

We can also consider the implications for a developed economy that increases its abatement effort.

$$z(t) = \frac{s(1-\theta)}{n+g+\delta} + \left[z(0) - \frac{s(1-\theta)}{n+g+\delta}\right]\exp(-\beta t) \quad (59)$$

$$\beta = (1-\alpha)(n+g+\delta). \quad (60)$$

- The growth rate of the capital-to-output ratio will be most negative directly after the policy change.
- $\Rightarrow$ The growth rate of output will be lowest directly after the policy change.
- $\Rightarrow$ The growth rate of pollution will be most negative directly after the policy change.

Is climate change different?

The problem of climate change sounds very familiar to models of fixed (finite) factors. Yet, we have overcome the Malthus poverty trap and the scarcity of other finite factors of production. Should we expect the same with climate change?

In the abstract, we can again overcome the scarcity problem by using green energies or abatement. Just as in the Solow model, this is something we can invest in (no longer a fixed factor) and theoretically in infinite supply (the sun).

What makes the problem more difficult are missing property rights. With other non-renewable resources, prices rise when the resource experiences shortage. With pollution, we have a *tragedy of the common*.

In the cases of water and air pollution, the problem may be solvable at the national level through taxes and regulations. However, green house gases require an international solution.

Is climate change different? II

The Nobel price winning economist Nordhaus warns in Nordhaus et al. (1992) to translate lessons from other non-renewables one-to-one to the case of green-house gases:

"Economists have often belied their tradition as the dismal science by downplaying both earlier concerns about the limitations from exhaustible resources and the current alarm about potential environmental catastrophe. However, to dismiss today's ecological concerns out of hand would be reckless. Because boys have mistakenly cried wolf in the past does not mean that the woods are safe."

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